Indian J. Phys. **73B** (1), 83-91 (1999)

IJP B
— an international journal

Dependence of the radial distribution function for azimuthal conductivity of arc plasma on tube radii

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Received 24 September 1998, accepted 3 November 1998

Abstract The distribution function for the radial variation of azimuthal conductivity of an arc plasma as proposed by Ghosal *et al* (1978) has been studied in four arc tubes of different radii The calculation of half-widths from the distribution function indicates that the plasma becomes less and less concentrated along the axis with the increase of the tube radius The value of axial conductivity has been calculated in four arc tubes and an analytical expression presented to represent the variation of axial conductivity with arc tube radius

Keywords Radial distribution function, azimuthal conductivity, arc plasma, radiofrequency current

PACS No. 52 70 Ds

1. Introduction

The radiofrequency non-immersive probe method has been utilised to study the variation of radial distribution profile of conductivity in a mercury arc by Ghosal *et al* [1, 2]. It has been shown that radiofrequency coil probe method in conjunction with the usual probe method may jointly provide useful information in connection with the structural behaviour of conductivity and electron density of the plasma. They have concluded, however, that the average azimuthal electrical conductivity of an arc plasma can be estimated by determining the change in band-width of a coil wound around the arc tube due to the presence of the plasma column within it. The method used in the present investigation as utilised by Ghosal *et al* [1] needs no calibration in contrast to the methods used by other authors like Mikoshiba and Smy [3] and Ciampi and Talini [4].

Ciampi and Talini [4] deduced expressions and measured two distinct average values of conductivity but did not, however, explore the structural behaviour of conductivity or electron density. With the help of a method based on the relation between the plasma parameters and the impedence of a radiofrequency coil placed co-axially around the plasma they obtained electron concentration and electron atom collision frequency for momentum transfer, assuming

Bessel type electron distribution along radius of the tube. Ghosal *et al* [2] questioned the Bessel distribution of charge density for an arc in the usual pressure range 10^{-2} to 10^{-3} torr and presented the new technique for obtaining the conductivity profile for a mercury arc plasma. The object of the present work is to find out and explain the dependence of the proposed distribution function for conductivity profile and their half-widths on tube radius.

2. Theoretical consideration

According to Ghosal *et al* [2], the azimuthal conductivity $\sigma_0(r)$ of the plasma at a distance r from the axis can be expressed as a polynomial expansion around $r = R$, and is written in the approximate form

$$\sigma(r) = \sigma_0 \,[1 - (r/R)^2]^n \,, \tag{1}$$

where σ_0 and n are to be determined and R is the radius of the tube. The quantity n can be determined experimentally and is given as

$$n = R^2/a - 2, \tag{2}$$

where $$a = [(\alpha - 1)/f^2k^2l\,](E/I)\,R_0 \,.$$

In eq. (2) α denotes the ratio of the radiofrequency current without and with the plasma of interest, f the frequency of the radiofrequency coil probe, l the length of the coil, k a constant depending upon the number of turns of the primary coil, R_0 the radiofrequency resistance, E the axial electric field strength per unit length and I is the discharge current.

Since

$$\int_0^R \sigma_0\,[1-(r/R)^2]^n\, r dr = I/2\,\pi E \,,$$

after integration, it has been deduced (Ghosal *et al* [2]) as

$$\sigma_0 = (I/E)\,2(n+1)/2\,\pi\,R^2 . \tag{3}$$

Though an expression has been provided for n in the radial distribution formula for conductivity, it is worthwhile to investigate whether the factor n depends on the radius of the arc tube. It is also to examine whether the distribution formula is valid irrespective of the radius of the arc tube and whether the axial conductivity becomes a function of the radius of the arc tube for the same input energy.

3. Experimental arrangement

The experimental arrangement for this work has been already reported by Ghosal *et al* [1]. Measurements have been made for a mercury arc plasma formed within the four arc tubes of approximately constant length 31–32 cm in which anode-cathode spacing is around 24 cm. The oscillator coil was placed closed to the work-coil wound around the discharge tube and the induced radiofrequency voltage was tuned with a variable condenser which forms a secondary tank circuit with the work-coil. The tuned currents were measured with the help of a radiofrequency milliammeter. The main discharge current has been changed by the high current capacity rheostats inserted in series with the DC supply. Any change in the tuned radiofrequency current should provide an indication of the change of resistive impedance of the resonant circuit. A number of measurements of tuned radiofrequency current, both in the presence and in the absence of plasma have been made for different discharge currents but at fixed exciting frequency 3.69 MHz. The circuit constants are given in Table 1.

Table 1. Circuit constants.

Parameter	Set I	Set II	Set III	Set IV
Outer diameter of tube in cm	1.22	1.85	2 15	2 64
Inner diameter of tube in cm	0 98	1.56	1 75	2 35
Coil length in cm	9.00	5.60	5 90	4 00
Coil diameter in cm	1.22	1 85	2 15	2 64
Wire diameter in cm	0 20	0 20	0 20	0.20
Probe-to-probe separation in cm	8.50	8.50	6 60	9 30
Number of turns in the coil	77	44	50	33
Radiofrequency resistance of the coil in Ω	14	11	19	15
Inductance of the coil in mH	10	10	19	16

The two tungsten probes within glass capsules with a bare tip of 0.1 cm have been utilised to measure the axial electric field strength in the plasma column.

4. Results and discussion

The values of $(\alpha - 1)$ as determined experimentally have been plotted against different arc currents (in the range of 2.25 A – 5A) for different tube radii in Figure 1. The variation is linear in nature for different values of tube radii.

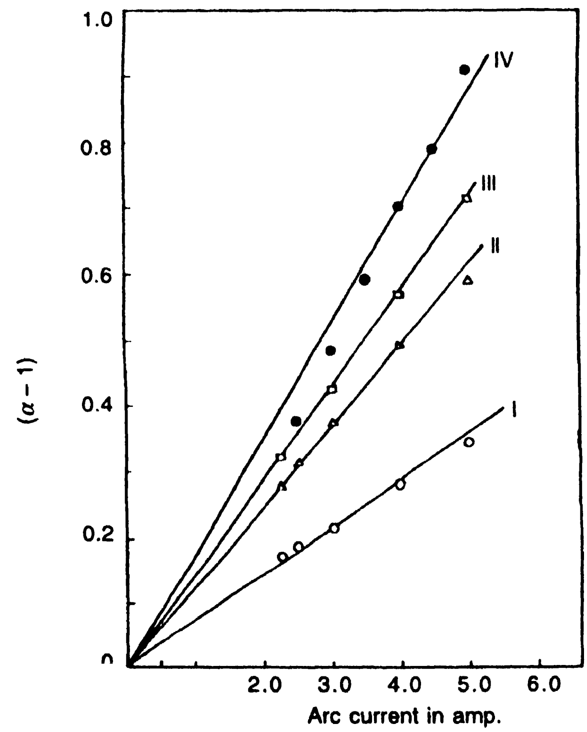

Figure 1. Variation of $(\alpha - 1)$ with arc current, I (R = 0 49 cm), II (R = 0.78 cm), III (R = 0.875 cm), IV (R = 1.185 cm).

As has been shown by Ghosal *et al* [1]

$$\alpha - 1 = \omega^2 M^2 / R_1 R_0 ,$$

where ω is the angular frequency of the applied radiofrequency field, M the mutual inductance, R_0 is the radiofrequency resistance of the coil and R_1 is the axial resistance of the plasma. For a particular tube radius, ω, M and R_0 are constants and hence $(\alpha - 1)$ is proportional to I/R_1 and hence to the arc current. As the values of M and R_0 will be different for different tube radii, the dependence of $(\alpha - 1)$ with arc current will vary with tube radius.

Values of n calculated from eq. (2) have been plotted in Figure 2 for different I/E values against different tube radii. It is observed that n has a tendency towards saturation for higher I/E, values and it becomes a function of R, the radius of the arc tube.

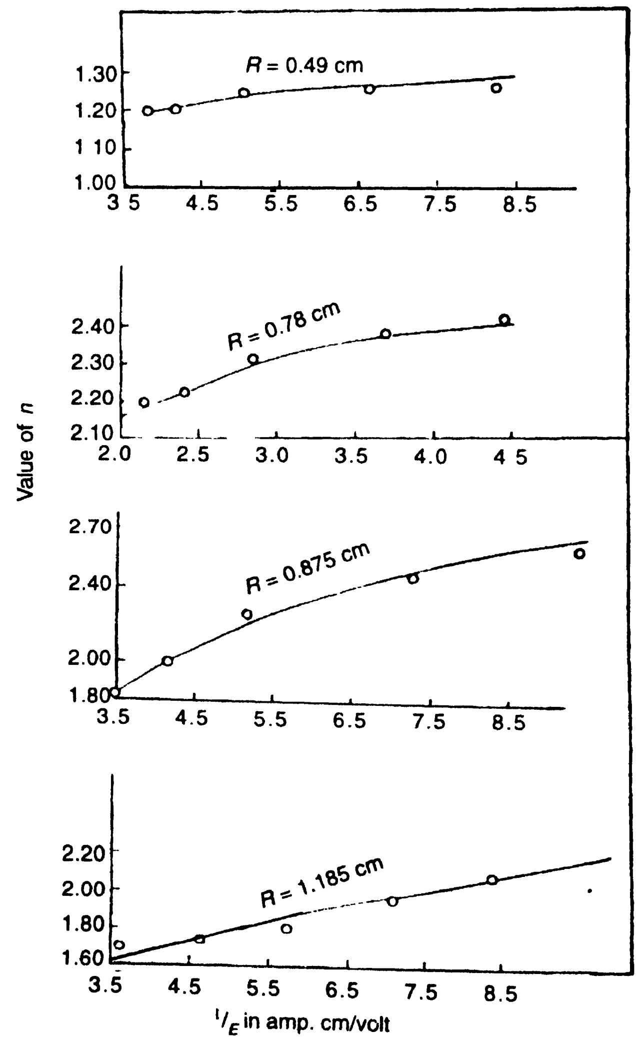

Figure 2. Variation of the value of n with I/E ; R as a parameter.

In order to find how the axial conductivity σ_0 varies with tube radius the values of σ_0 have been calculated from eq. (3) and plotted against I/E in Figure 3. In all the cases σ_0 increases linearly with I/E but with increasing slope as tube radius is reduced.

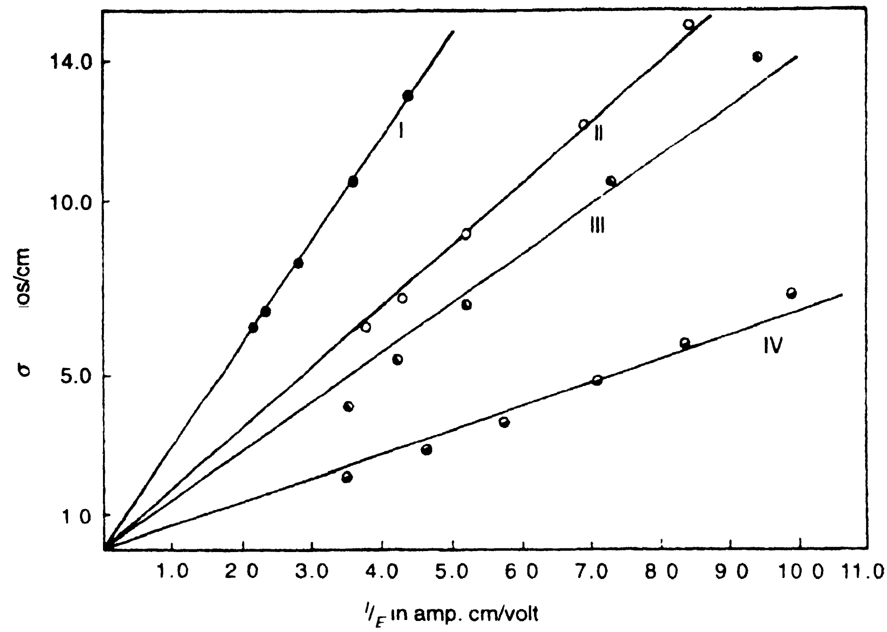

Figure 3. Variation of σ_0 with I/L at different tube radii I (R = 0 49 cm), II (R = 0 78 cm), III (R = 0 875 cm), IV (R = 1.185 cm)

The radial variation of $\sigma(r)$ from $r = 0$ to $r = R$ has been calculated for different tube radii for some specific values of I/E one of which is shown in Figure 4. From the nature of the

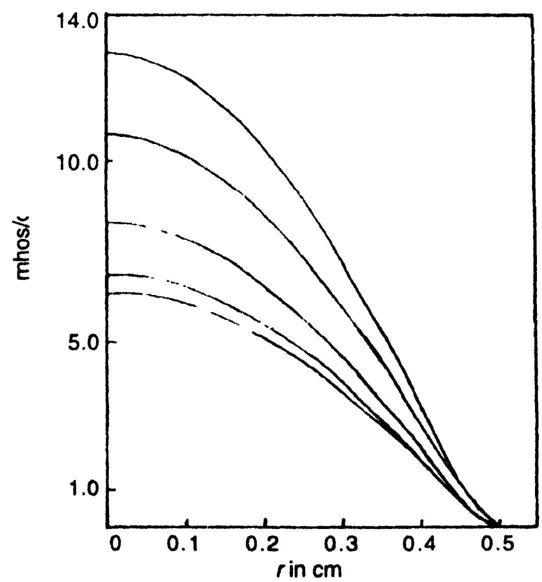

Figure 4. Electrical conductivity distribution as function of r for I/E values (amp.cm/volt) 4.4, 3.6, 2.8, 2.36 and 2.186 respectively from top downwards in the arc tube of radius 0 49 cm.

representative curve it is evident that not only the conductivity at the axis shows a rapid increase with the increase of the arc current but at the same time the nature of the distribution of azimuthal conductivity undergoes a remarkable change indicating that the discharge becomes more and more constricted with the increase of the arc current. The nature of variation is, however, the same in arc tubes of different radii indicating that the proposed distribution formula (eq. 1) is valid for tube of any radius.

To study the variation of radial conductivity profile for different tube radii the value of $\sigma(r)$ has been calculated against tube radii for some values of *I/E*. In this case also, one of the representative curves is shown in Figure 5. It is evident from the figure that for each value of *I/E* the axial conductivity is largest for tube with smallest radius and gradually decreases with the increase in tube radius. The radial conductivity falls much more rapidly for the tube with small radius whereas the rate of fall is much less for the tubes with larger radii. The case is almost analogous with the selectivity of radio circuit with smaller and smaller resistance. The rate of dimunition of the radial conductivity thus depends upon the tube radius which indicates that n in eq. (1) is an explicit function of the tube radius as has been observed here independently.

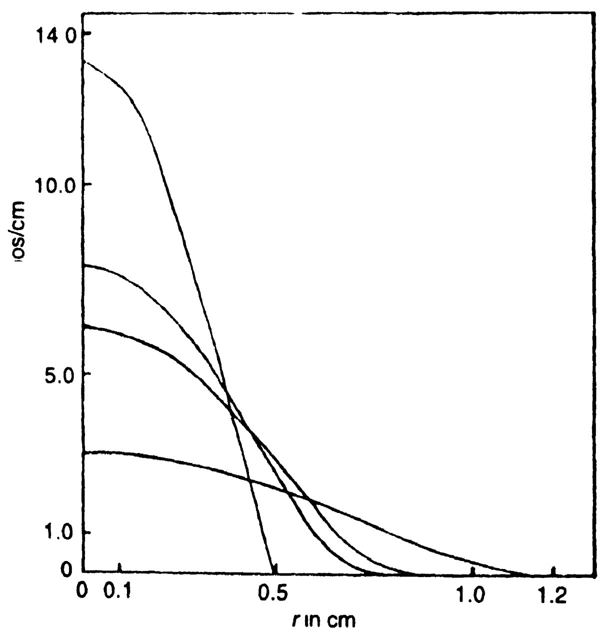

Figure 5. Electrical conductivity distribution as function of r for R values (cm) 0 49, 0 78, 0 875 and 1.185 respectively from top downwards at fixed *I/E* value (4.5 amp.cm/volt).

If $\hat{r}$, a particular value of r, denotes the half-width of the conductivity distribution curve, then we get

$$\hat{r} = R\,[1-(1/2)^{1/n}]^{1/2}$$

Taking the value of n from Figure 2 for $I/E = 4.5$ amp.cm/volt, half-widths for different tube radii have been plotted in Figure 6. The half-width construed as a measure of the constriction of the plasma column increases rapidly with the increase of the tube radius for a fixed value of *I/E*.

The values of σ_0, the axial conductivity have been plotted against the tube radii for tubes used in the present investigation for I/E values namely 4.5, 4.0 and 3.5 amp.cm/volt and

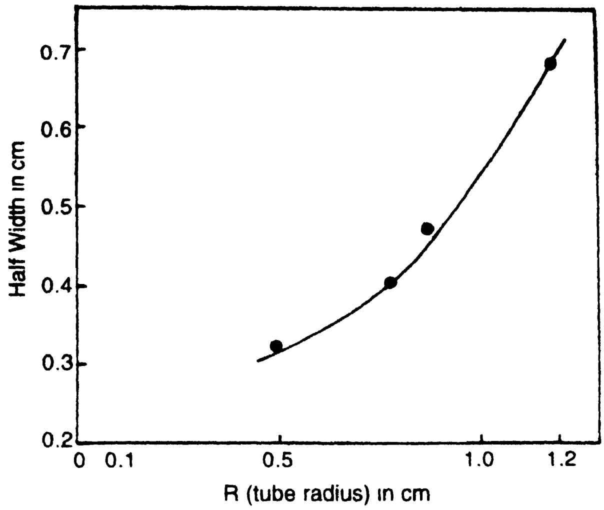

Figure 6. Variation of half-widths of the conductivity distribution function with R, tube radius at fixed I/E value (4.5 amp cm/volt)

are shown in Figure 7. The curves show that σ_0 decreases exponentially with the increase of R. Alternatively, values of log σ_0 have been plotted against R in Figure 8 and the graph shows a linear variation with R. Thus the variation of σ_0 with R can be represented by an equation of the form

$$\sigma_0(R) = \sigma_{0<R\to 0>} \exp(-\beta R),$$

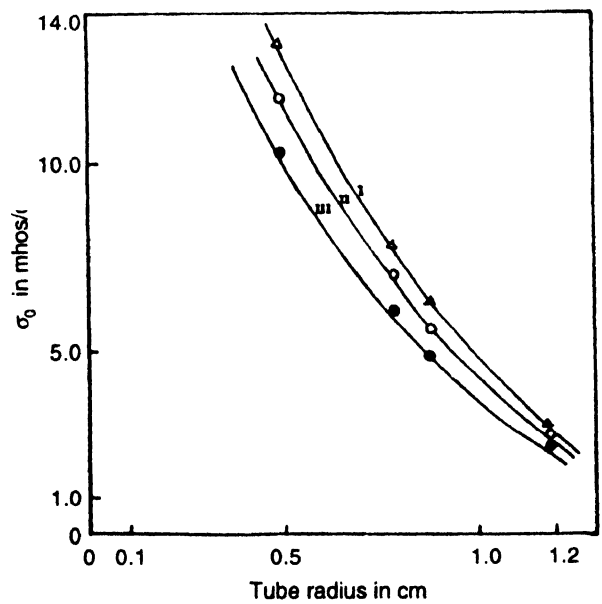

Figure 7. Variation of σ_0 with tube radius I(I/E = 4.5 amp cm/volt) II (I/E = 4.0 amp.cm/volt), III (I/E = 3.5 amp.cm/volt).

where β is a constant and $\sigma_{0<R\to 0>}$ the axial conductivity at $R \to 0$. We can calculate the value of β by a statistical method, which is shown here for I/E = 4.5 amp.cm/volt as :

$$\sigma_0(R) = \sigma_{0<R\to 0>} \exp(-\beta R),$$

$$\log_e \sigma_0(R) = \log_e \sigma_{0<R\to 0>} - \beta R,$$

$$S = \Sigma[\log_e \sigma_0(R) - \log_e \sigma_{0<R\to 0>} + \beta R]^2,$$

$$dS/d\beta = 2\Sigma R[\log_e \sigma_0(R) - \log_e \sigma_{0<R\to 0>} + \beta R] = 0,$$

$$\Sigma R \log_e \sigma_0(R) = \log_e \sigma_{0<R\to 0>} \Sigma R - \beta \Sigma R^2,$$

$$\beta = \log_e \sigma_{0<R\to 0>} \Sigma R - \Sigma R \log_e \sigma_0(R) / \Sigma R^2,$$

$$\log_e \sigma_{0<R\to 0>} = 3.65 \quad \Sigma R = 3.33 \quad \Sigma R^2 = 3.01835,$$

$$\Sigma R \log_e \sigma_0(R) = 5.80589 \quad \beta = 2.103.$$

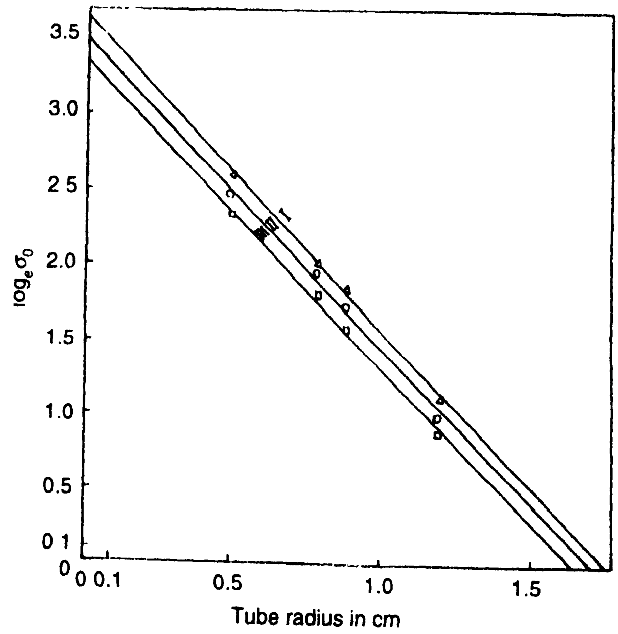

Figure 8. Variation of $\log_e \sigma_0$ with tube radius I(I/E = 4.5 amp.cm/volt) II(I/E = 4.0 amp cm/volt), III (I/E = 3.5 amp cm/volt)

The calculated value of β for I/E = 4.0 and 3.5 amp.cm/volt has been found to be 2.091 and 2.079 respectively. Thus, it is observed that β decreases with decrease of I/E *i.e.* input energy.

5. Conclusions

In a mercury vapour tube, the arc completely occupies the tube for low currents but as the current is increased, the arc column contracts and the light becomes more intense at the axis of the tube. Besides this, with the reduction of the tube radius the plasma column also contracts. It is also established that the value of on-axis conductivity decreases exponentially with tube radius. It has also been observed that the conductivity at the axis increases with the decrease of the tube radius for the same value of *I/E*. This may be attributed to the fact that the gas density is higher in a tube of smaller areas for the same value of pressure and consequently the rate of collision becomes higher which results in greater ionization and hence to increased conductivity.

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